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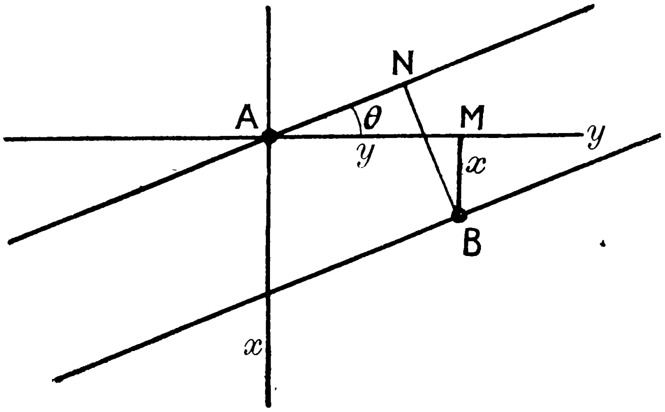
A Method of Double-Star Measurement. By J. B. Dale, M.A.

1. The method of double-star measurement in general use consists of a direct determination of the polar coordinates of one of the stars relative to the other. The operations involved are, theoretically at any rate, very simple, but the comparison of the same observer's observations made on different occasions, or of the observations of different observers, reveals the existence of well-defined systematic errors both in distance and position angle. The cause of these errors is somewhat obscure, and their elimination is difficult and uncertain.

It is, however, a well-established principle that the best method of eliminating systematic error of any sort is to vary as far as possible the conditions of measurement, and the object of the following note is to suggest a method of measurement in which such variation is made. It is possible that results obtained in this way may serve as a check upon the results obtained by the ordinary method of procedure, and may throw some light on the source and character of the errors involved.



2. In the figure, A and B represent two stars whose relative position is to be determined. Take A as an origin of rectangular coordinates, the positive direction of the x axis being drawn towards the north and the y axis being the parallel. Then (x, y) being the coordinates of B, the distance of B from a line drawn through A making a positive angle θ with Ay is

$$x \cos \theta + y \sin \theta = \text{BN}.$$

This distance may be measured in the usual way with the micrometer, one wire A′ being set on A and the other B′ on B.

If a is the reading of B′ when it coincides with A′, then the reading when on B is

$$z = x \cos \theta + y \sin \theta + a.$$

Theoretically, the three unknowns x, y, a can be found from measurements made with three different values of θ, but of course it is preferable to increase the number of settings and deduce the

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values by a least square solution of the resulting equations. I have found it convenient to assign to θ the values 0°, 30°, 60° . . . . 330°, and so obtain twelve values of z, z_1, z_2 . . . . z_{12}. The values of x and y may then be found in the usual manner.

3. The labour of solution is much simplified by arranging the work as follows:—

	z_1	z_2	z_3	z_4	z_5	z_6
	z_7	z_8	z_9	z_{10}	z_{11}	z_{12}
Sums	s_1	s_2	s_3	s_4	s_5	s_6
Differences	d_1	d_2	d_3	d_4	d_5	d_6

	d_1	d_2	d_3	d_4
		d_6	d_5	
Sums		d_2+d_6	d_3+d_5	
Differences		d_2-d_6	d_3-d_5	

Then

$$x=\tfrac{1}{6}\left[d_1+\sqrt{\tfrac{3}{2}}(d_2-d_6)+\tfrac{1}{2}(d_3-d_5)\right]$$
$$y=\tfrac{1}{6}\left[d_4+\sqrt{\tfrac{3}{2}}(d_3+d_5)+\tfrac{1}{2}(d_2+d_6)\right]$$
$$a=\tfrac{1}{12}\Sigma(s)$$

also

$$\rho=\sqrt{x^2+y^2} \quad \text{and} \quad \phi=\tan^{-1}(y/x).$$

In a perfect set of measurements the s's would be identical. Further, we should find

$$\begin{array}{rcccc} d_1 & : & \sqrt{\tfrac{3}{2}}(d_2-d_6) & : & \tfrac{1}{2}(d_3-d_5) \\ =d_4 & : & \sqrt{\tfrac{3}{2}}(d_3+d_5) & : & \tfrac{1}{2}(d_2+d_6) \\ =2 & : & 3 & : & 1 \end{array}$$

If the residuals of the z's be formed, and ϵ be the mean error of the z's, the mean error of x, y or ρ is $\epsilon/\sqrt{6}$ and of a, $\epsilon/\sqrt{12}$. The mean error in ϕ is $\epsilon/(r\sqrt{6})$ radians or $(23{\cdot}4\ \epsilon/r)^\circ$.

4. The following set of measures of γ Leonis made on May 23, 1909 will illustrate the procedure. The unit is one division of the micrometer screw head, and is equal to 0″·4208. The telescope has an aperture of $3\tfrac{1}{2}$ inches.

		Sums.	Diffs.
$z_1 = 50{\cdot}4$	$z_7 = 58{\cdot}9$	109·3	− 8·5
$z_2 = 54{\cdot}3$	$z_8 = 53{\cdot}8$	108·1	+ 0·5
$z_3 = 59{\cdot}9$	$z_9 = 49{\cdot}6$	109·5	+ 10·3
$z_4 = 62{\cdot}7$	$z_{10} = 46{\cdot}2$	108·9	+ 16·5
$z_5 = 64{\cdot}0$	$z_{11} = 46{\cdot}0$	110·0	+ 18·0
$z_6 = 61{\cdot}8$	$z_{12} = 47{\cdot}2$	109·0	+ 14·6

$$\begin{array}{rl@{\qquad\qquad}rl} d_1 = & -\ 8{\cdot}5 & d_4 = & 16{\cdot}5 \\ \sqrt{\tfrac{3}{2}}(d_2-d_0) = & -\ 13{\cdot}2 & \sqrt{\tfrac{3}{2}}(d_3+d_5) = & 24{\cdot}6 \\ \tfrac{1}{2}(d_3-d_5) = & \underline{-\ \ 3{\cdot}9} & \tfrac{1}{2}(d_2+d_6) = & \underline{\ 7{\cdot}6} \\ x = & -\ 4{\cdot}27 & y = & +8{\cdot}12 \end{array}$$

whence $\rho = 9{\cdot}174 = 3''{\cdot}86$, $\phi = 117^\circ{\cdot}8$.

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The residuals $z_o - z_c$ are as follows:—

+·1	−·6	+·4	0	+·2	−·5
+·1	−·4	−·1	−·3	+·6	+·4

The mean error in z is 0·32 = 0″·135.
The mean error in x, y, ρ is 0·13 = 0″·055.
The mean error in ϕ is . . 0°·82.
The correction for the parallel was negligible.

5. It is not yet possible to state any definite results obtained by this method, but measures of the same stars made on different nights suggest the following:—

(1) The mean error in distance derived from one complete set of measures is fairly constant, the average being 0·25 of a division.
(2) The mean discordance between measures made on different nights is somewhat larger; it averages 0·30 of a division, and appears to increase with the distance.
(3) The mean error of the derived values of the position angle derived from one night's measures is about twice as large as the mean discordance between measures on different nights.
(4) The residuals frequently show a periodic run, those between 0° and 180° being repeated between 180° and 360°. They could be well represented by terms of the form

$$a \cos 2\theta + b \sin 2\theta.$$

These results are in agreement with the general experience of observers, but it is rather remarkable that measures on different nights lead to much smaller alterations in the ratios of the z's than the differences in their absolute values would lead one to expect. There seems to be an alteration in scale value. The discordances are larger than slight variations in adjustment for focus could produce, but the periodic run in the residuals leads one to think that it may be a parallactic effect, due to the stars and wires not being together in focus.

King's College, London:
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Magnitude of η Argus, 1909. By R. T. A. Innes.

The comparison stars were, as in former years :—

C. G. A. Cluster Catalogue No. 121,	8·00 mag.	8 colour.
Gilliss 1332, ...	7·60 ,,	4 ,,

The separate observations are :—

1909 Mar. 31	7·75 mag.	5 colour.
Apl. 11	7·8 ,,	6 ,,
,, 22	7·8 ,,	6 ,,

A low power on the Grubb 9-in. refractor was used. The means of the observations by years, which I have made since 1896, are as follows :—

	Mag.	Colour.	No. of Obs.
1896·4	7·58	5·0	4 – 1
1899·5	7·71	7·0	4 – 2
1900·3	7·68	6·3	3
1901·3	7 78	6·8	3
1902·1	7·72	6·5	3
1905·5	7·67	7·5	5 – 2
1908·4	7·75	5·7	4 – 3
1909·3	7·78	5·7	3

These observations show no variation since 1899 greater than the errors of estimation.

Government Observatory, Johannesburg :
1909 *April* 24.